

***A*nnual *P*hosphorus *L*oss *E*stimator**

**Theoretical Documentation**

**Version 3.0**

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**Introduction**

Non-point source pollution of fresh waters by phosphorus (P) can accelerate eutrophication and limit water use for drinking, recreation, and industry. Because agriculture is a non-point source of P to surface waters, there is a need to quickly and accurately identify fields prone to excessive P loss and also management practices to reduce P loss. Process-based simulation models can assess agricultural P loss, but their data and expertise requirements prohibit their use as routine management tools. A simpler approach used throughout the U.S. and in Europe is a field-scale P Index. However, most P Indexes do not explicitly quantify a mass of P loss (e.g., lb/ac/yr). Instead, they use readily obtainable information in an additive and/or multiplicative framework to calculate a qualitative risk of P loss, typically expressed as low, medium, high, or very high. A tool that reliably quantifies field-scale P loss but remains easy to use and requires only readily obtainable inputs is an alternative to process-based models and qualitative P Indexes.

A field-scale P loss quantification tool offers attractive characteristics for P loss reduction planning. It could be used in any region where the simulated P loss processes dominate. Because a quantification tool can be formally validated with measured data, it can be designed to accurately account for the relative effect of different management practices on P loss and describe the forms of P lost (i.e., dissolved or sediment-bound). Thus, tradeoffs among various management practices can be assessed, such as the variable effect of no-till management on decreasing sediment P loss but increasing dissolved P loss from surface applications of manure or fertilizer. A quantification tool can provide information about seasonal trends, such as P loss from snow-melt runoff or due to significant precipitation variations. Such information should in-turn drive better management decisions, such as if fall or winter manure application represents the greater risk of P loss given the nature of runoff potential. Fourth, a P loss quantification tool will entail computer automation of inputs and calculations, which offers several opportunities. Soil and climate databases, and erosion, runoff, soil, and crop models can be combined with the P loss tool while still maintaining simple user inputs. This would allow P loss estimation across climate years, crop rotations, or a range of management and fertilization practices. Quantified assessments of the ability of different management practices to decrease P loss could also be linked to optimization programs to design a suite of management practices that balance P loss with other farms goals, such as economic viability. Alternatively, output could be linked to measurable water quality goals.

For many agricultural fields, the dominant P transport pathway is surface runoff; and the dominant P sources are soil, manure, and fertilizer. For these situations, a P loss quantification tool must estimate soil erosion and particulate P loss, runoff, and dissolved P loss from soil, manure, and fertilizer. This document describes a simple, user-friendly tool that has been developed to estimate field-scale P loss in surface runoff from agricultural fields.

In spring of 2022, the **APLE** model was modified (APLE 3.0) to include soil bulk density as a model input. In previous versions, a value of 1.2 g cm-3 was assumed; but Bolster et al. (2022) showed that predictions of Mehlich-3 STP were quite sensitive to values of soil bulk density. **APLE 3.0** also allow users to estimate runoff using the Curve Number method; previous versions required runoff to be input by the model user. Finally, **APLE 3.0** allows users to define error ranges for model inputs, and using Monte Carlo simulations, the updated version of APLE provides estimates of model prediction uncertainties based on assigned uncertainties in model inputs. Further details on incorporating model and data uncertainties in evaluating model performance can be found elsewhere (Bolster and Vadas, 2013; Bolster et al., 2016, 2022).

**APLE Model Description**

The **APLE** model is a Microsoft Excel spreadsheet model that runs on an annual time step. The model simulates sediment bound and dissolved P loss in surface runoff. It does not consider subsurface loss of P through leaching to groundwater or artificial drainage networks. However, the model does simulate downward movement of soil P from the first soil layer to the second and out of the second layer to more accurately maintain a realistic soil P balance over time. This downward movement of P is not included as a model output. The model is intended to simulate edge-of-field P loss for uniform fields of several hectares in size, or smaller. **APLE**does not consider the impact on P loss of grassed waterways or buffers that may occur beyond the field edge. The model considers different kinds of animal manure (beef, dairy, poultry, swine), applied either by machine or by grazing beef or dairy cattle, but considers only highly soluble commercial fertilizers such as superphosphate, triple superphosphate, or mono- and di-ammonium phosphate.

**APLE** is intended to be user-friendly and does not require extensive input data to operate. All data are input directly into the spreadsheet (See **APLE**User’s Manual). User-input data include:

* Soil property data, including depth of the top two soil layers, Mehlich-3 soil test P, soil clay content, and soil organic matter content
* The area of the field (ha)
* The annual rain, runoff, and erosion amounts
* The total annual crop P export in harvested material
* When grazing animals are present, the total number of animal days in the field, including beef cattle and calves, dairy lactating and dry cows, and dairy heifers and calves.
* For manure applications, the manure amount applied, manure % solids, manure total P205 content, % of manure total P that is water extractable P, the % of manure that is incorporated, and the depth of incorporation.
* For fertilizer applications, the mass of fertilizer P applied, the % of fertilizer that is incorporated, and the depth of incorporation.

**DISSOLVED PHOSPHORUS RUNOFF FROM MANURE AND FERTILIZER**

**APLE** estimates annual dissolved P loss from surface manure and fertilizer based on the daily time-step models of Vadas et al. (2007; 2004; 2008). In **APLE**, manure is applied in either a solid form or a liquid form, as specified by the user. Fertilizer is assumed to be applied in a solid form. Based on limited data of Vadas (2006), **APLE** assumes that for any manure with solids content less than 15%, 60% of applied manure P infiltrates into soil immediately at application and becomes unavailable for direct loss in runoff. **APLE** also assumes that the solids from these liquid manures remaining on the soil surface after the initial infiltration cover only 50% of the field area. If liquid manure is injected without tillage, as specified by the user, APLE assumes that a maximum of 90% of applied manure P will be injected into soil at low rates of liquid (e.g., 1000 gallons/acre), with injected amounts decreasing to a minimum of 60% (i.e., the same as non-injected liquid) at 25000 gallons/acre). This is done because high rates of liquid application are not able to be effectively injected and will leave manure P on the soil surface. If tillage occurs, **APLE** incorporates any applied manure or fertilizer according to user-specified depths of incorporation and percentages of P applied that are incorporated. **APLE** estimates annual dissolved P loss directly from any manure or fertilizer remaining on the soil surface.

For any manure applied, the model assumes a portion of the manure total P is in a water extractable form. Both the manure total P content and the percent of total P that is WEP are user-specified variables. For the **APLE** model, manure WEP should be measured by shaking fresh manure with de-ionized water at a water to solids extraction ratio of 250:1 for 1 h, filtering extracts through 0.45-um filters, and measuring P in filtrates (Vadas et al., 2004). Manure WEP is commonly estimated at extraction ratios other than 250:1. For example, the Arkansas pasture P Index uses manure WEP to estimate field-scale, annual P loss, but bases WEP values on a 10:1 extraction ratio. However, data generated from other extraction ratios can be converted to a 250:1 equivalent using relationships from Vadas et al. (2005a). The model estimates the amount of dissolved manure P loss in runoff from the manure WEP on the soil surface

The portion of manure P that is not in a WEP form (non-WEP) at application can mineralize during the year and add to the amount of manure WEP on the soil surface. **APLE** assumes that for winter-applied manure, which APLE simulates as the first season of the year, 20% of non-WEP left on the soil surface after infiltration of liquid P, injection, or tillage mineralizes into WEP. This value is 15% for spring-applied manure, 10% for summer-applied manure, and 5% for fall-applied manure. The user specifies the season of application.

The user can also specify how many dairy or beef cattle graze the field during the year. This will add manure and manure P to the field and increase the amount of dissolved manure P loss in runoff. **APLE** assumes daily feces production and fecal total P content for dairy and beef cattle as listed in Table 1. Feces WEP at deposition is 55% of total P, and 75% of feces WEP is available the same year for P loss in runoff and 25% is available the following year. **APLE** also assumes that 20% of feces non-WEP on the soil surface mineralizes into WEP the same year.

Table 1. Daily feces production and fecal total P content for grazing dairy and beef cattle.

|  |  |  |
| --- | --- | --- |
| Animal Type | Daily Fecal Production (kg) | Fecal Total P content (kg/kg) |
| Lactating Dairy Cow | 8.9 | 0.0088 |
| Dairy Heifer | 3.7 | 0.0054 |
| Dairy Dry Cow | 4.9 | 0.0061 |
| Dairy Calf | 1.4 | 0.0054 |
| Beef Cow | 6.6 | 0.0067 |
| Beef Calf | 2.7 | 0.0092 |

**APLE** estimates annual manure or fertilizer dissolved P loss in runoff as:

Manure Runoff P = (Manure WEP)(Annual Runoff/Precipitation)(P Distr. Factor) [1]

Fertilizer Runoff P = (Fertilizer P) (Annual Runoff/Precipitation) (P Distr. Factor) [2]

The P Distribution Factor is an empirical factor between 0.0 and 1.0 that distributes released P between runoff and infiltration and is calculated as:

Manure: P Distribution Factor = (Runoff/Precipitation) 0.225 [3]

Fertilizer: P Distribution Factor = 0.034 exp [(3.4) (Runoff/Precipitation)] [4]

The precipitation represents total rain, snow, and irrigation for an entire year. For fall-applied manure, APLE assumes 75% of manure WEP on the soil surface is available for loss in runoff the same year of application and 25% the following year.

When applying equation [1] and [2] for liquid manure and grazing dung, **APLE** reduces the amount of dissolved P loss in runoff by a factor that accounts for the fact that these manures and dung do not cover the entire soil surface and not all the annual precipitation interacts with them to leach P. In calculating the reduction factor for grazing dung, **APLE** first assumes that each 250 g of dung (dry weight) covers an area of 659 cm2 (James et al., 2007) and calculates what percentage of the field area this covers. **APLE** then calculates the dung reduction factor as:

Reduction Factor = 1.2 x (250 x % cover) / [(250 x % cover) + 73.1)] [5]

where % cover is expressed in a decimal form. For liquid manures, **APLE** assumes % cover is 0.5, and uses equation [6] to determine the P loss reduction factor.

Reduction Factor = 2.2 x (250 x % cover) / [(250 x % cover) + 300.1)] [6]

Equations [5] and [6] are taken from the daily time-step model of Vadas et al. (2007), where they are used to calculate the portion of manure P that is released for a given storm.

**SEDIMENT BOUND AND DISSOLVED PHOSPHORUS RUNOFF FROM SOIL**

**APLE** estimates sediment P loss in runoff as:

Sediment P Loss = (Eroded Sediment)(Soil Total P)(P Enrichment Ratio)(10-6) [7]

where:

Sediment P Loss: Annual P loss in runoff associated with eroded sediment (kg ha-1)

Eroded Sediment: Annual soil lost in runoff due to erosion (kg ha-1)

Soil Total P: Total P content of surface soil (mg kg-1)

P Enrichment Ratio: Unitless ratio of total P in eroded sediment to that in the source soil

**APLE** calculates the P Enrichment Ratio based on equations from Menzel et al. (1980) and Sharpley (1980):

ln (P Enrichment Ratio) = 2.2 - 0.25 ln (eroded sediment) [8]

Soil total P in Eq. [7] is estimated as described in the soil P processes section of this document.

**APLE** estimates dissolved inorganic P loss in runoff (kg ha-1) from soil using the equation of Vadas et al. (2005b):

Dissolved Soil Runoff P = (Soil Labile P) (0.005) (Annual Runoff) (10-6) [9]

where Annual Runoff is in L ha-1. Soil Labile P (mg kg-1) is estimated as described in the soil processes section. The 0.005 value is an extraction coefficient that estimates dissolved P in runoff (mg L-1) from soil Labile P.

**SOIL PHOSPHORUS PROCESSES**

***Number of Soil Layers***

**APLE** is intended to simulate processes in only the topsoil but can simulate two layers in the topsoil. This is intended to estimate P stratification (i.e., significantly different P concentrations in different soil layers) in soils with no or limited tillage. This would be important for pastures or no-till soils where regular P additions to soil might cause more P to accumulate in the top 1 inch of soil than deeper in the topsoil. The depth of the two soil layers is specified by the user at the beginning of a simulation.

***Soil Phosphorus Pools and Exchanges***

The APLE model does all soil P process calculations in the units of kg ha-1. Soil P routines for each topsoil layer in **APLE** are based on the model of Jones et al. (1984) and simulate three inorganic P pools (Labile, Active, and Stable) and one Organic P pool. Labile P represents easily desorbable P immediately available for plant uptake or transfer to runoff, and is defined as P extracted by anion exchange resin (Sharpley et al., 1984). Labile P is initialized based on user-specified concentrations of Mehlich-3 soil test P (ppm), with **APLE** assuming that Labile P is one half the value of Mehlich-3 P (Vadas and White, 2010). Active P represents more stable P that is not easily desorbable, but in equilibrium with Labile P. Active P is initialized from Labile P and a P sorption coefficient, or P availability Index, (PSP) as:

Active P = (Labile P) (1 - PSP) / PSP [10]

The PSP represents how much of any inorganic P added to soil remains Labile P upon reaching relative equilibrium. A PSP of 0.4 means 40% of added P remains Labile P and 60% becomes Active P. Experimentally, PSP values are determined by measuring Labile P in a soil, adding inorganic P to the soil, incubating the soil for six months, and again measuring Labile P. The percentage of added P that remains Labile is the PSP (Sharpley et al., 1984). In **APLE**, PSP is estimated from user-defined soil properties of clay content (%) and organic matter content (%) as (Vadas and White, 2010):

PSP = -0.053\*ln (% clay)+0.001\*(Labile P) - 0.029\*(% Organic C) + 0.42 [11]

The organic carbon (C) content is assumed to be 58% of user-defined organic matter content (%). The PSP is given lower and upper limits of 0.05 and 0.90.

Soil Stable P is assumed to be four times the size of Active soil P. Soil Organic P is initialized from user-defined soil organic C amounts and by assuming that the C:Nitrogen (N) ratio of soil organic substances is 14:1 and the N:P ratio is 8:1. This method for estimating Organic P results in similar estimates as equations for estimating soil organic P from Sharpley et al. (1984). **APLE** maintains this ratio of organic P to organic C ratio as organic P fluctuates from either addition of manure or in mineralization (see two following paragraphs). **APLE** estimates soil total P (which is used for sediment P loss in runoff in Eq. [5]) as the sum of the Labile, Active, Stable, and Organic P pools.

When P is added to soil in manure or fertilizer, **APLE** first distributes the added P to the appropriate soil layer based on user-defined tillage practices, depths, and degree of soil mixing by the tillage operation. **APLE** assumes that 5% of added manure P becomes Organic P. This 5% represents a final amount of manure P that would remain organic after all annual mineralization processes are complete. The remaining 95% of added manure P and all added fertilizer P are added to the soil inorganic P pools. APLE distributes added inorganic P between the Labile, Active, and Stable pools based on the equilibrium relationships established by the daily time-step model of Jones et al. (1984). In that model, all added P is initially added to the Labile P pool, which disturbs the equilibrium between the two pools as described in Eq. [8]. The P is thus slowly added to the Active P pool at a rate of 0.1 per day. Moving P from Labile P to Active P in turn disturbs the equilibrium between Active P and Stable P, and P is moved from Active to Stable P. Based on this model of Jones et al., APLE calculates what fraction of added inorganic ultimately ends up in the Stable P pool as:

Fraction Added P to Stable P = (-0.187 x PSP) + 0.189 [12]

The remainder of added inorganic P is distributed between Labile and Active P based on the PSP value, which determines the relative size of the pools at equilibrium.

When annual P removal from a soil layer is greater than annual P inputs, **APLE** decreases soil P from the three inorganic soil P pools. Based on the model of Vadas et al. (2006), **APLE** uses Eq. [13] to determine the fraction of P that is removed from the Labile P pool:

Fraction P Removed from Labile P = 0.41 x PSP2 +0.54 x PSP + 0.005 [13]

The remaining P decrease is partitioned between the Active and Stable pools based on their relative sizes. For example, if a soil layer loses 10 kg ha-1 of P in a year and has a PSP of 0.3, then 2.06 kg ha-1 of P is removed from the Labile pool. If Stable P is four times Active P, then 1.59 kg ha-1 of P is removed from the Active pool and 6.35 kg ha-1 of P is removed from the Stable pool.

**APLE** estimates soil organic P mineralization if Labile P becomes less than 7.5 mg kg-1 byallowing enough organic P to mineralize to maintain Labile P at 7.5 mg kg-1. Any organic P mineralized is moved from the Organic P pool to the Labile P pool. Mineralization also occurs if there is net decrease in soil Labile P. In this instance, P mineralized is equal to 15% of the calculated decrease.

***Soil Mixing Between Topsoil Layers***

APLE mixes P between the two topsoil layers based on the user-defined degree of soil mixing based on tillage or natural mixing processes, such as mixing by earthworms or freeze-thaw actions. If one soil layer contains more P than the other, the overall effect is to reduce P in one layer and increase it in the other by an amount proportional to the degree of mixing.

***Phosphorus Leaching from Topsoil Layers***

**APLE** estimates the fraction of annual precipitation that leaches through the two topsoil layers in L ha-1 as:

Leachate/Precipitation = -0.07 x ln(Soil Layer Depth) + 0.6 [13]

where soil layer depth is the depth of the bottom of the soil layer in inches. This equation is based on data from Nelson et al (2005), who measured the amount of water leaching through a sandy soil in North Carolina.

APLE estimates a concentration of dissolved P (mg L-1) in the soil leachate based on a phosphorus sorption isotherm, which relates the amount of P sorbed on the soil and the amount dissolved in the soil water. This is similar to the approach taken by Nelson and Parsons (2006) to modify the GLEAMS model to better simulate P leaching in waste-amended soils. In APLE, P sorbed onto the soil (mg kg-1) and dissolved P in soil water (mg L-1) are related as:

P Sorbed = (a) ln(Dissolved P) + b [14]

In **APLE**, P sorbed is assumed to be equal to the sum of soil Labile P at the beginning of the year and half of the added manure and fertilizer P that are estimated to remain Labile P by the end of the year. **APLE** calculates the a and b variables as:

a = (173.51) (% soil clay) + 8.48 [15]

b = (4.726) (a) – 8.97 [16]

Equations [14]-[16] are taken from Vadas (2001). **APLE** sets a maximum dissolved P concentration of 20 mg L-1 for soil leachate based on observations of Nelson et al. (2005) and a maximum amount of P (kg ha-1) that can be leached equal to P Sorbed in Eq. [14]. A portion of P that leaches from the first layer is added to the Labile P in the second layer and a portion leaves the modeled system. The portion added to the second layer is determined according to the relative thickness of the two topsoil layers as:

Portion of P into Second Layer = exp [-0.2 x (1st Layer Thickness /

2nd Layer Thickness) [17]

Phosphorus that leaches from the second layer leaves the modeled system.

***Crop Export of Phosphorus***

**APLE** accounts for soil P export in harvested crops (crop P removal) according to the user-specified annual amount. APLE assumes a portion of P exported by crops comes from the two simulated soil layers depending on the depth of the simulated layers as:

P uptake from soil layer = 0.2367\*ln(soil depth)-0.1184 \* Total Crop P Uptake [18]

where soil depth is in cm and Total Crop Uptake is specified by the user (Fan et al., 2016). Crop P removal from the soil is limited to the sum of P in the Labile and Active pools, a portion of the Stable pool, as well as a portion of P added in manure and fertilizer.

**PREDICTING ANNUAL RUNOFF FROM DAILY PRECIPITATION DATA**

**APLE 3.0** allows users topredict annual runoff from daily (or event) precipitation data using the SCS Curve Number method. This empirical method, which is the most widely used method for calculating rainfall excess, was originally developed for use in small rural watersheds (NRCS, 1986). While limitations to this method have been noted, including its empirical nature and being developed for predicting streamflow at the watershed scale (Hawkins, 2014;Ogden et al., 2017), it is still widely used for predicting field-scale overland runoff because it only requires a single input variable, and no simple alternative to the method exists (Hawkins et al., 2020). The general form of the model is:

for R > *Ia* ; *Q* = 0 for *R* < *Ia* [19]



where *Q* is runoff depth, *R* is rainfall, *Ia* is the initial abstraction (usually assumed to be 0.2S), and *S* is the potential retention calculated from the curve number, CN, by:

[20]



when *R* and *Q* are in mm. The CN is assumed to be a function of soil group, land cover, and antecedent moisture condition. Published tables of CNs are available for a wide range of soil types and land uses/covers (USDA-NRCS, 2004). In keeping with the simple approach of APLE, runoff is calculated from daily precipitation, rather than event precipitation, consistent with the original development and calibration of the CN method (Hawkins et al., 2019). To allow for greater flexibility with the model, the user can also manually enter runoff rates as determined by other means.

**INCORPRATING MODEL INPUT UNCERTAINTY INTO MODEL PREDICTIONS**

**APLE 3.0** give users the option to estimate prediction uncertainties based on user-defined errors in model inputs. Specifically, the user now has the option of assigning an error range of ± 0, 5, 10, 15, 20, 25, 30, 35, or 40% for each model input from a drop-down menu. These ranges represent the assumed minimum and maximum range of realistic input values. The error ranges can be determined from variability in measured values or reported in the literature when available; otherwise, best-professional judgement can serve as a guide to estimate reasonable error ranges in the model inputs.

Based on the assigned range, values for the input parameters are randomly chosen from a symmetrical beta distribution using the Excel What-If Analysis tool in the Data tab to perform the Monte Carlo Simulations (MCS). The beta distribution is defined as:

[21]



where *α* and *β* are the shape parameters; to obtain a symmetrical distribution, both parameters are assigned a value of 2. For each MCS (n=1000), input parameter values are obtained using the BETA.INV(p, α, β, min, max) Function, which returns the inverse of the beta cumulative probability density function, where p is the probability (randomly assigned using the Rand Function) and min and max represent the lower and upper bounds, respectively, of the distribution. The lower and upper bounds are calculated from the input parameter value and the user-defined error range. The advantage to using a bounded distribution such as the beta distribution is that it prevents the possibility of randomly selecting an unrealistic value during the MCS as may occur when generating values from a normal probability distribution function. The symmetrical beta distribution is similar in shape to the normal distribution but with a lower and wider peak; thus, the range in highly probable values will be wider for the beta distribution compared with the normal distribution.

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